Qualitative Spatial Reasoning about Relative Position

The Tradeoff between Strong Formal Properties and Successful Reasoning about Route Graphs

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Abstract. Qualitative knowledge about relative orientation can be expressed in form of ternary point relations. In this paper we present a calculus based on ternary relations. It utilises finer distinctions than previously published calculi. It permits differentiations which are useful in realistic application scenarios that cannot directly be dealt with in coarser calculi. There is a price to pay for the advanced options: useful mathematical results for coarser calculi do not hold for the new calculus. This tradeoff is demonstrated by a direct comparison of the new calculus with the flip-flop calculus.

Keywords: Qualitative Spatial Reasoning, Cognitive Modelling, Robot Navigation

1 Introduction

Qualitative Spatial Reasoning (QSR) abstracts from metrical details of the physical world and enables computers to make predictions about spatial relations, even when precise quantitative information ist not available [Cohn, 1997]. From a practical viewpoint QSR is an abstraction that summarizes similar quantitative states into one qualitative characterization. A complementary view from the cognitive perspective is that the qualitative method *compares* features within the object domain rather than by *measuring* them in terms of some artificial external scale [Freksa, 1992]. This is the reason why qualitative descriptions are quite natural for humans.

The two main directions in QSR are topological reasoning about regions [Randell et al., 1992], [Renz and Nebel, 1999] and positional (orientation and distance) reasoning about point configurations [Freksa, 1992], [Clementini et al., 1997], [Zimmermann and Freksa, 1996], [Isli and Moratz, 1999]. More recent approaches in QSR that model orientations are [Isli and Cohn, 2000], [Moratz et al., 2000]. For robot navigation, the notion of path is central [Latombe, 1991] and requires the representation of orientation and distance information [Röfer, 1999]. Since we are especially interested in qualitative calculi suitable for robot navigation we developed a positional calculus

C. Freksa et al. (Eds.): Spatial Cognition III, LNAI 2685, pp. 385–400, 2003.

for this task. The calculus is based on results of psycholinguistic research on reference systems. We compare the new calculus with the simpler flip-flop calculus. We can demonstrate that even if the flip-flop calculus has stronger formal properties the new calculus is better suited for certain applications in robot navigation.

2 Qualitative Representation of Relative Position

Positional calculi are influenced by results of psycholinguistic research in the field of reference systems. These findings are presented by Thora Tenbrink in the article of Moratz, Tenbrink, Fischer, and Bateman in this volume [Moratz et al., 2002]. The results point to three different options to give qualitative descriptions of spatial arrangements of objects which are labeled by Levinson [Levinson, 1996] as *intrinsic*, *relative,* and *absolute*.

We can find examples for all three options of reference systems in the QSR literature. An intrinsic reference system was used in the dipole calculus [Schlieder, 1995], [Moratz et al., 2000]. Relative reference systems in QSR were introduced by Freksa [Freksa, 1992]. Andrew Frank's cardinal direction calculus corresponds to an absolute reference system [Frank, 1991], [Ligozat, 1998].

Qualitative position calculi can be viewed as computational models for projective relations in relative reference systems. To model projective relations (like "left", "right", "front", "back") in relative reference systems, all objects are mapped onto the plane $\mathcal{D}$. The mapping of an object O onto the plane $\mathcal{D}$ is called $p_{\mathcal{D}}(O)$. The center μ of this area can be used as point-like representation O' of the object O: $O' = \mu(p_{\mathcal{D}}(O))$. Using this abstraction we will henceforth consider only point-like objects in the 2D-plane.

Figure 1 shows a simple model for the left/right-dichotomy in a relative reference system given by *origin* and *relatum* (corresponing to Levinsons terminology). Origin and relatum define the reference axis. It partitions the surrounding space in a left/right-dichotomy. The spatial relation between the reference system and the *referent* is then described by naming the part of the partition in which the referent lies. In the configuration depicted in Figure 1 the referent lies to the *left*[1] of the relatum as viewed from the origin.

This scheme ignores configurations in which the referent is positioned on the reference axis. Freksa [Freksa, 1992] used a partition that splits these configurations into three sets: the referent then is either behind the relatum, at the same position like the relatum or in front of the relatum. Ligozat [Ligozat, 1993] subdivided the arrangements with the relatum or in front of the relatum in the cases where the referent is between relatum and origin, at the same position as the origin, or behind the origin. We obtain then the partion shown on Figure 2. Ligozat calls this calculus the flip-flop calculus. For a compact notation we use abbreviations as relation symbols.

For A, B, and C as origin, relatum, and referent, Figure 3 shows point configurations and their qualitative descriptions, respectively. Isli and Moratz [Isli and Moratz, 1999] introduced two additional configurations in which origin and relatum have exactly the

[1] The natural language terms used here are meant to improve the readability of the paper. For issues of using QSR representations for modeling natural language expressions please refer to the article of Moratz, Tenbrink, Fischer and Bateman in this volume [Moratz et al., 2002].

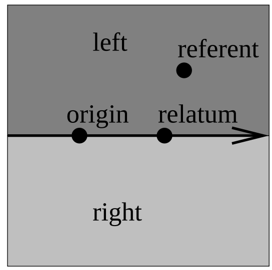

Fig. 1. The left/right-dichotomy in a relative reference system

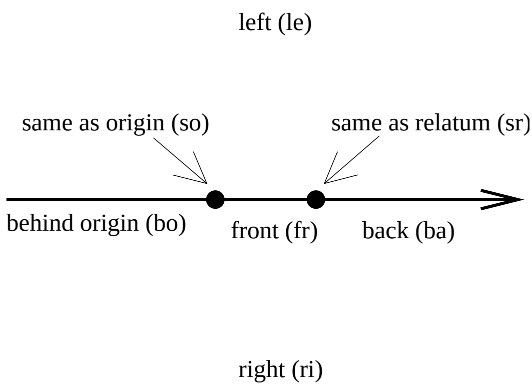

Fig. 2. Adding relations for referents on the reference axis

same location. In one of the configurations the referent has a different location, this relation is called **dou** (for double point). The configuration with all three points at the same location is called **tri** (for triple point). A system of qualitative relations which describe all the configurations of the domain and do not overlap is called jointly exhaustive and pairwise disjoint (JEPD).

The simple flip-flop calculus models “front” and “back” only as linear acceptance regions. Vorwerg et al. [Vorwerg et al., 1997] showed empirically that a cognitive adequate model for projective regions needs acceptance regions for “front” and “back” which have a similar extent as “left” and “right”. Freksa’s single cross calculus [Freksa, 1992] has this feature (see Figure 4). The front region consists of “left/front” and “right/front”, the left region consists of “left/front” and “left/back”. The intersection of both regions models the left/front relation.

The calculus we will now present is derived from the single cross calculus but makes finer distinctions. These finer distinctions are motivated by the application scenario dealing with route graphs presented at the end of our paper. The partition of the calculus is shown in Figure 5.

The letters f, b, l, r, s, d, c stand for front, back, left, right, straight, distant, close, respectively. The terms front, back, etc. are given for mnemonic purposes. The use of the TPCC relations in natural language applications is shown in this volume in an

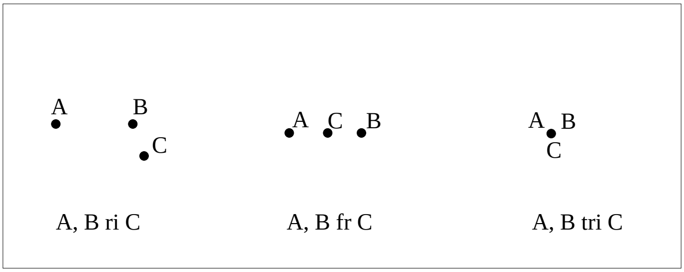

Fig. 3. Examples of point configurations and their expressions in the flip-flop calculus. We use an infix notation where the reference system consisting of origin and relatum is in front of the relation symbol and the referent is behind the relation symbol.

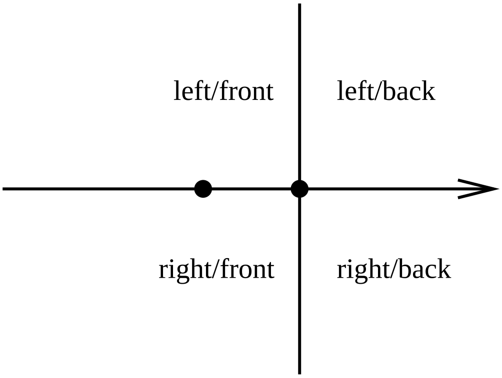

Fig. 4. The single cross calculus

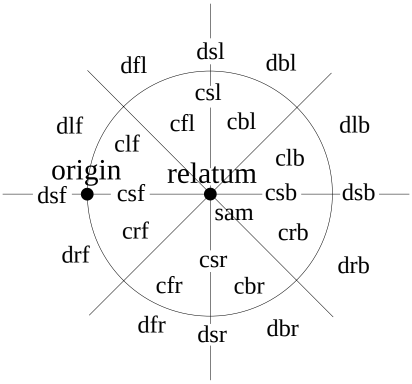

Fig. 5. The reference system used by the TPCC calculus

article by Moratz, Tenbrink, Fischer and Bateman [Moratz et al., 2002]. They use the TPCC relations for natural human robot interaction. The configuration in which the referent is at the same position as the relatum is called **sam** (for “same location”). The two special configurations in which origin and relatum have the same location **dou**, **tri** are also base relations of this calculus. This system of qualitative spatial relations and the inference rules described in the next section is called *Ternary Point Configuration Calculus* (TPCC). To give a precise, formal definition of the relations we describe the corresponding geometric configurations on the basis of a Cartesian coordinate system represented by $\mathbb{R}^2$. First we define the special cases for $A = (x_A, y_A)$, $B = (x_B, y_B)$ and $C = (x_C, y_C)$.

$$
\begin{aligned}
A, B \text{ dou } C \quad &:= \quad x_A = x_B \wedge y_A = y_B \wedge (x_C \neq x_A \vee y_C \neq y_A) \\
A, B \text{ tri } C \quad &:= \quad x_A = x_B = x_C \wedge y_A = y_B = y_C
\end{aligned}
$$

For the cases with $A \neq B$ we define a relative radius $r_{A,B,C}$ and a relative angle $\phi_{A,B,C}$:

$$
\begin{aligned}
r_{A,B,C} \quad &:= \quad \frac{\sqrt{(x_C - x_B)^2 + (y_C - y_B)^2}}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}} \\
\phi_{A,B,C} \quad &:= \quad \tan^{-1} \frac{y_C - y_B}{x_C - x_B} - \tan^{-1} \frac{y_B - y_A}{x_B - x_A}
\end{aligned}
$$

Then we have the following spatial relations:

$$
\begin{aligned}
A, B \text{ sam } C \quad &:= \quad r_{A,B,C} = 0 \\
A, B \text{ csb } C \quad &:= \quad 0 < r_{A,B,C} < 1 \wedge \phi_{A,B,C} = 0 \\
A, B \text{ dsb } C \quad &:= \quad 1 \leq r_{A,B,C} \wedge \phi_{A,B,C} = 0 \\
A, B \text{ clb } C \quad &:= \quad 0 < r_{A,B,C} < 1 \wedge 0 < \phi_{A,B,C} \leq \pi/4 \\
A, B \text{ dlb } C \quad &:= \quad 1 \leq r_{A,B,C} \wedge 0 < \phi_{A,B,C} \leq \pi/4 \\
A, B \text{ cbl } C \quad &:= \quad 0 < r_{A,B,C} < 1 \wedge \pi/4 < \phi_{A,B,C} < \pi/2 \\
A, B \text{ dbl } C \quad &:= \quad 1 \leq r_{A,B,C} \wedge \pi/4 < \phi_{A,B,C} < \pi/2 \\
A, B \text{ csl } C \quad &:= \quad 0 < r_{A,B,C} < 1 \wedge \phi_{A,B,C} = \pi/2 \\
A, B \text{ dsl } C \quad &:= \quad 1 \leq r_{A,B,C} \wedge \phi_{A,B,C} = \pi/2 \\
A, B \text{ cfl } C \quad &:= \quad 0 < r_{A,B,C} < 1 \wedge 1/2\,\pi < \phi_{A,B,C} < 3/4\,\pi \\
A, B \text{ dfl } C \quad &:= \quad 1 \leq r_{A,B,C} \wedge 1/2\,\pi < \phi_{A,B,C} < 3/4\,\pi \\
A, B \text{ clf } C \quad &:= \quad 0 < r_{A,B,C} < 1 \wedge 3/4\,\pi \leq \phi_{A,B,C} < \pi \\
A, B \text{ dlf } C \quad &:= \quad 1 \leq r_{A,B,C} \wedge 3/4\,\pi \leq \phi_{A,B,C} < \pi \\
A, B \text{ csf } C \quad &:= \quad 0 < r_{A,B,C} < 1 \wedge \phi_{A,B,C} = \pi \\
A, B \text{ dsf } C \quad &:= \quad 1 \leq r_{A,B,C} \wedge \phi_{A,B,C} = \pi \\
A, B \text{ crf } C \quad &:= \quad 0 < r_{A,B,C} < 1 \wedge \pi < \phi_{A,B,C} \leq 5/4\,\pi \\
A, B \text{ drf } C \quad &:= \quad 1 \leq r_{A,B,C} \wedge \pi < \phi_{A,B,C} \leq 5/4\,\pi
\end{aligned}
$$

$$
\begin{aligned}
A, B \text{ cfr } C &:= 0 < r_{A,B,C} < 1 \wedge 5/4\,\pi < \phi_{A,B,C} < 3/2\,\pi \\
A, B \text{ dfr } C &:= 1 \leq r_{A,B,C} \wedge 5/4\,\pi < \phi_{A,B,C} < 3/2\,\pi \\
A, B \text{ csr } C &:= 0 < r_{A,B,C} < 1 \wedge \phi_{A,B,C} = 3/2\,\pi \\
A, B \text{ dsr } C &:= 1 \leq r_{A,B,C} \wedge \phi_{A,B,C} = 3/2\,\pi \\
A, B \text{ cbr } C &:= 0 < r_{A,B,C} < 1 \wedge 3/2\,\pi < \phi_{A,B,C} < 7/4\,\pi \\
A, B \text{ dbr } C &:= 1 \leq r_{A,B,C} \wedge 3/2\,\pi < \phi_{A,B,C} < 7/4\,\pi \\
A, B \text{ crb } C &:= 0 < r_{A,B,C} < 1 \wedge 7/4\,\pi \leq \phi_{A,B,C} < 2\,\pi \\
A, B \text{ drb } C &:= 1 \leq r_{A,B,C} \wedge 7/4\,\pi \leq \phi_{A,B,C} < 2\,\pi
\end{aligned}
$$

There are cases in which we only have coarser spatial knowledge or in which we are at the border of a segment of the partition and cannot decide safely due to measurement errors. Then we use sets of the above defined relations to denote disjunctions of relations. Figure 6 shows a situation where it is not sensible to decide visually between the alternatives $A, B\,\mathrm{clb}\,C$ and $A, B\,\mathrm{cbl}\,C$. Such a configuration is described by the relation $A, B\,(\mathrm{cbl}, \mathrm{clb})\,C$.

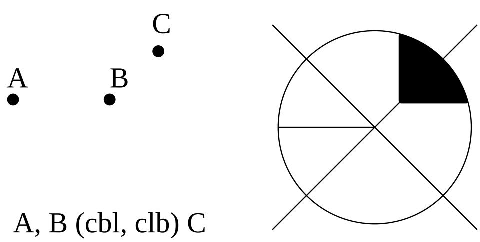

Fig. 6. Coarser spatial knowlege

3 Deductive Reasoning about Relative Positional Information

In the last section we defined relations between triples of points on the 2D-plane. Now we define a set of unary and binary operations that allow to deduce new relations about point sets from given relations about these points. Unary operations (transformations) use relations about three points to deduce a relation which holds for a permuted sequence of the same points. Binary operations (compositions) deduce information from two relations which have two points in common (the set consists of four points). The result then is a relation about one of the common points and the two other points.

3.1 Permutations

Because we have three arguments, we have 3! = 6 possible ways of arranging the arguments for a transformation. Following Zimmermann and Freksa

[Zimmermann and Freksa, 1996] we use the following terminology and symbols to refer to these permutations of the arguments (a,b : c):

term	symbol	arguments
identical	ID	a,b : c
inversion	INV	b,a : c
short cut	SC	a,c : b
inverse short cut	SCI	c,a : b
homing	HM	b,c : a
inverse homing	HMI	c,b : a

The transformation tables for the flip-flop calculus are presented in Isli and Moratz [Isli and Moratz, 1999]. We therefore present here only the transformation table for the TPCC calculus on table 8. In contrast to the flip-flop calculus the TPCC calculus is not closed under the transformations. That means that results of a transformation can constitute proper subsets of the base relations. Since we need many sets of relations as results of transformed relations we introduce here an iconic notation of the relations which makes the presentation more compact:

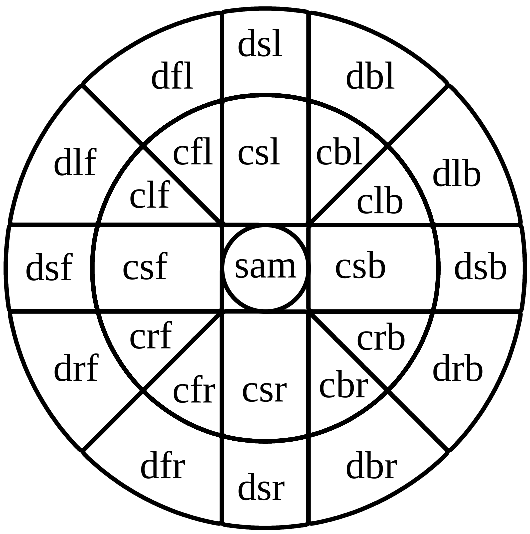

Fig. 7. Iconic Representation for TPCC-Relations

The segments corresponding to a relation are presented as filled segments. Unions of relation then simply have several segments filled. The reference axis and the dividing lines between left, right, front and back are also presented in the icon to make the visual identification of the relation symbol easier. The iconic representation is easier to translate into its semantic content (the denoted spatial point configuration) compared with a representation that uses the textual relation symbol. And unions can be expressed in a compact way.

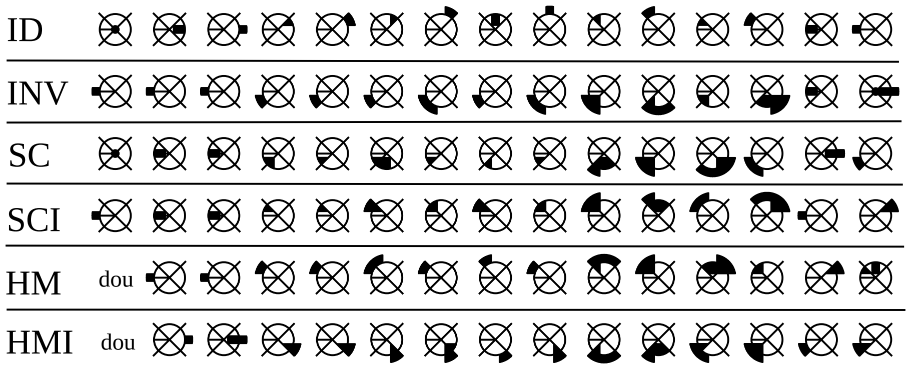

Fig. 8. Permutation Table for TPCC-Relations

In order to reduce the size of the table trivial cases for **dou** and **tri** are omitted. Symmetric cases can be derived using a reflection operation (reflection on an axis). The results of SC(**dsf**) and SCI(**dsf**) also include **dou** as a result.

3.2 Composition

With ternary relations, one can think of different ways of composing them. However there are only a few ways to compose them in a way such that we can use it for enforcing local consistency [Scivos and Nebel, 2001]. In trying to generalize the path-consistency algorithm [Montanari, 1974], we want to enforce 4-consistency [Isli and Cohn, 2000]. We use the following (strong) composition operation:

$$\forall A, B, D : \; A, B\, (r_1 \diamond r_2)\, D \;\leftrightarrow\; \exists C : \; A, B\, (r_1)\, C \wedge B, C\, (r_2)\, D$$

The composition table for the flip-flop calculus is presented in Isli and Moratz [Isli and Moratz, 1999].

Unfortunately, the TPCC calculus is not closed under strong composition. For that reason we can not directly enforce 4-consistency. But we can define a weak composition operation $r_1 \Diamond r_2$ of two relations r_1 and r_2. It is the most specific relation such that:

$$\forall A, B, D : \; A, B\, (r_1 \Diamond r_2)\, D \;\leftarrow\; \exists C : \; A, B\, (r_1)\, C \wedge B, C\, (r_2)\, D$$

While using the weak composition we can not enforce 4-consistency we still get usefull inferences. We use this weak composition for inferences in the application scenario in section 4.

The table for weak composition of TPCC relations is shown in figure 9. The first operand determines the row, the second operand the column. Again the table omits entries which can be found by reflection in order to reduce the size of the table. And the trivial cases for **dou** and **tri** are omitted.

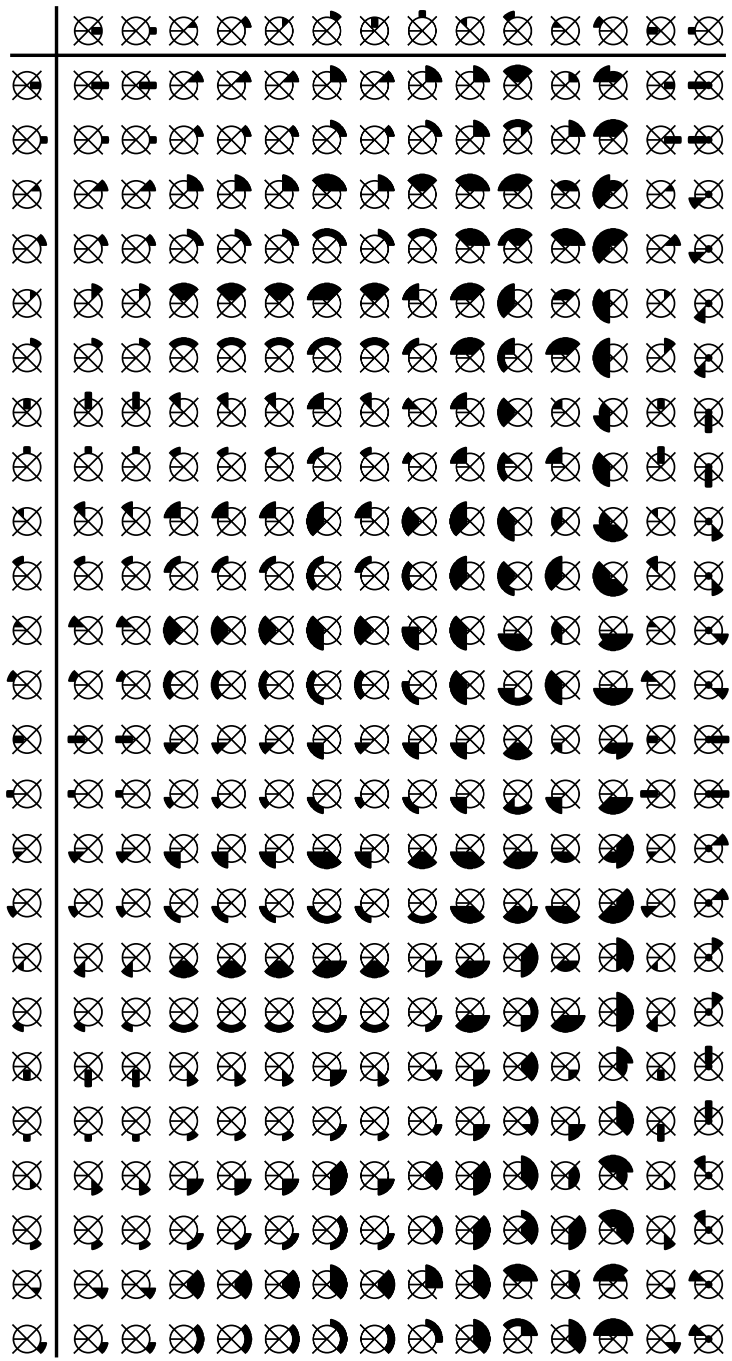

Fig. 9. Composition of TPCC-Relations

3.3 Constraint-Based Reasoning

The standard method for reasoning with relation algebras is to use Ladkin and Reinefeld's algorithm [Ladkin and Reinefeld, 1992] that uses backtracking employing the path-consistency algorithm as forward checking method. This scheme was extended by Isli and Cohn [Isli and Cohn, 2000] for ternary relation algebras. It can then easily be applied to the flip-flop calculus.

A prerequisite to using the standard constraint algorithms is to express the calculi in terms of relation algebras in the sense of Tarski [Ladkin and Maddux, 1994]. But since the TPCC-Calculus is not closed under the transformations and under the composition we can not use this scheme. However, simple path-based inferences can be performed using the following scheme. The two last relations of a path are composed. Then the reference system is incrementally moved towards the beginning of the path in form of a backward chaining.

For the detection of cyclic paths a reference system consisting of a path segment in the middle of the path is appropriate. Then the relative position of the points in both directions is derived and compared using an inversion operation (see appendix A for an example).

It can be proven that reasoning with the TPCC relations is in PSPACE. The idea of the proof sketch is as follows. The algebraic semantics of the relations implies that reasoning problems in the TPCC calculus can be expressed as equalities over polynomials with integer coefficients. Systems of such equalities can be solved using polynomial space [Renegar, 1992].

4 Path-Based Reasoning in Route Graphs

The flip-flop calculus and the TPCC calculus can be used to integrate local and survey knowledge about the spatial environment of an agent. Local knowledge can be sensorically acquired from one fixed point in space. Survey knowledge is an abstraction that integrates a number of local perceptions into a coherent whole. The local perceptions are typically acquired in a sequence during an exploration process. The accumulated local assessments of qualitative configurations have local frames of reference. The integration process needs to reason about the position of the salient objects in a global reference. Path integration can serve as a means to achieve the accumulation of local orientation information. The problem of detecting cyclic paths in a route graph is a sample application which we present to compare the coarse and the finer calculus in a typical application scenario.

From a qualitative viewpoint a path can be viewed as a sequence of qualitative positions. The positions are discriminated with respect to the environment. Therefore both calculi can be used to describe and to reason about paths. Qualitative positional reasoning about paths is used for robot navigation in the approach of Sogo and in the approach of Musto [Sogo et al., 1999] [Musto et al., 1999]. In our sample application the environment consists of a route graph [Werner et al., 1998]. The task is to derive a global map from locally perceived information. Reasoning from perceived local spatial arrangements about the underlying global layout of an environment is a form of abductive reasoning [Remolina and Kuipers, 2001] [Shanahan, 1996].

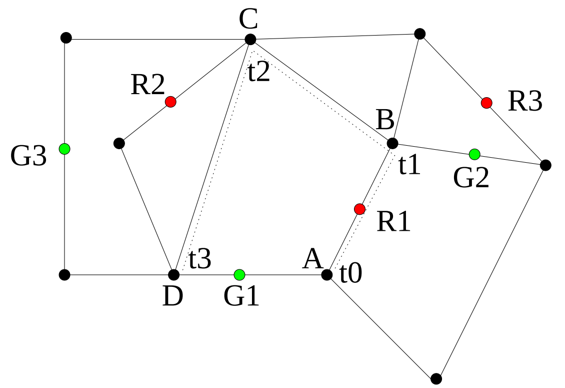

Fig. 10. A route graph

We focus here on a deductive subproblem. The problem is to decide whether two landmarks reached during an exploration can be identical due to a cyle in the path. The observations are collected at timepoints t0, t1, t2 and t3 (see figure 10). The local observations are expressed in both calculi. Then we test which landmarks G1 perceived from A, G2, G1 perceived from t3 and R1, R2 can be deduced to be distinct. The observations and the deductive inferences are listed in appendix A.

The result is that both calculi can deduce that G1 and G2 are distinct. Both calculi are correct and therefore do not deduce that G1 perceived from A and G1 perceived from t3 are distinct. But only the TPCC calculus can deduce that R1 and R2 are distinct. Using the same reasoning scheme the TPCC calculus can also deduce that R1 and R3 are distinct which needs not only orientation but also distance-based reasoning. This example shows that differentiations which are useful in realistic application scenarios are supported by the new TPCC calculus. These finer (but still coarse) distinctions can not be dealt with in the mathematically more elegant flip-flop calculus.

There are applications in which even finer qualitative acceptance areas are helpful. The techniques described here can still be used. But there is obviously no way to design icons that can express these finer distinctions. And the computation of the composition table can become difficult. Then an approximation of the composition results can be used. The possibility to use even finer qualitative distinctions can be viewed as a stepwise transition to quantitative knowledge which is the topic of the next subsection.

4.1 Comparison with a Quantitative Approach for Interval-Based Spatial Reasoning

The simplest and most common strategy to deal with coarse knowledge is to treat it as if it were precise metrical knowledge. Then the user has to rely on his good luck that all derived conclusions are valid. Compared to that unsafe approach qualitative spatial reasoning is *safe* because it only derives correct information as long as the input information as correct. This technical argument for QSR leads to the question whether QSR is the only way to do safe spatial reasoning. In scalar or one-dimensional domains

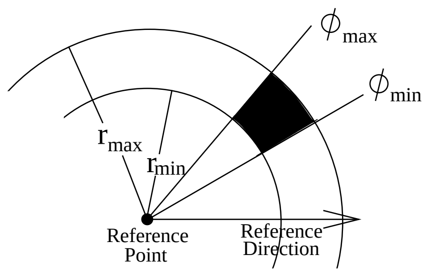

Fig. 11. A distance/orientation-interval and its parameters

interval-based reasoning serves the purpose of a safe quantitative alternative to qualitative reasoning. Therefore we now present a straightfoward quantitative approach which is based on distance/orientation-intervals.

A distance/orientation-interval (DOI) uses a point and a reference direction as anchor and has four additional parameters r_{min}, r_{max}, ϕ_{min} and ϕ_{max} (see figure 11).

These quantitative intervals can be propagated along pathes analogous to the qualitative counterparts. The respective reference directions then are determined by adjacent points on the path. The technical details of the interval propagation can be found in [Moratz, *in preparation*] . The quantitative calculus can solve all presented problems of the last section about the route graph when the observed intervals are sufficiently small.

Now we look at an example were we need a more expressive calculus. In the route graph depicted on figure 12 an agent travels from D to E via A, B, C. We model the perception of the agent like in the previous example. The agent can only perceive locations to which a direct straight link exits. Then it can not use the propagation of measured intervals to distinguish between D and E if the distance between D and E is sufficiently small.

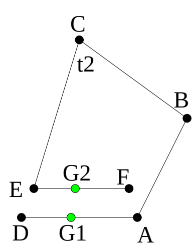

Fig. 12. Reasoning about the absence of features

We need to represent the information that seen from D to A there is no road junction at a direction differing from A. Because QSR can be seen as reasoning about space within first order logic [Isli and Cohn, 2000] [Renz and Nebel, 1999] we have *negation*, *disjuction* and *conjunction* already built in. So we can use the TPCC-Calculus to express our knowledge about the absence of a feature:

$$\forall_{x\in J, g\in G} \neg \left((g, D \bullet x) \wedge \mathrm{cn}\,(E, g) \wedge \mathrm{cn}\,(E, x) \right)$$

The symbol **cn** stands for the predicate *connected* (via a direct straight link). J ist the set of all road junctions, G is the set of all green landmarks. Adding this logical constraint to the observations we can distingish the road junctions D and E. We can not express this in the quantitative calculus because we have no logical operations. To extend a quantitative calculus in that direction is not a trivial task and would make it much more complex.

5 Conclusion and Perspective

We presented the new TPCC calculus for representing and reasoning about qualitative relative position information. We identified a system of 27 atomic relations between points and computed the composition table based on their algebraic semantics, which allows to apply constraint-based reasoning methods. It was demonstrated that reasoning with the TPCC relations is in PSPACE. Potential applications of the calculus are demonstrated with a small navigation example in route graphs.

In a comparison with a coarser calculus known in the literature we noticed that helpful mathematical properties are unfortunately not satisfied by the TPCC calculus. It is a matter of further studies how the framework of constraint satisfaction especially with respect to path consistency can be transferred to the new calculus.

Acknowledgement

The authors would like to thank Amar Isli, Jochen Renz, Alexander Scivos and Thora Tenbrink for interesting and helpful discussions related to the topic of the paper. And we would like to thank Sven Kröger for computing the composition table. This work was supported by the DFG priority program on Spatial Cognition.

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Appendix A: Inferences for the Route Graph Example

First we use the flip-flop calculus for representation and reasoning in the route graph example. We have the following observations at timepoints t:

$$t0: \quad e2(B), e1(A) \quad \text{ri} \quad g1(G1) \tag{1}$$
$$t1: \quad e3(C), e2(B) \quad \text{ri} \quad e1(A) \tag{2}$$
$$t1: \quad e1(A), e2(B) \quad \text{ri} \quad g2(G2) \tag{3}$$
$$t2: \quad e2(B), e3(C) \quad \text{le} \quad e4(D) \tag{4}$$
$$t3: \quad e3(C), e4(D) \quad \text{le} \quad g3(G1) \tag{5}$$

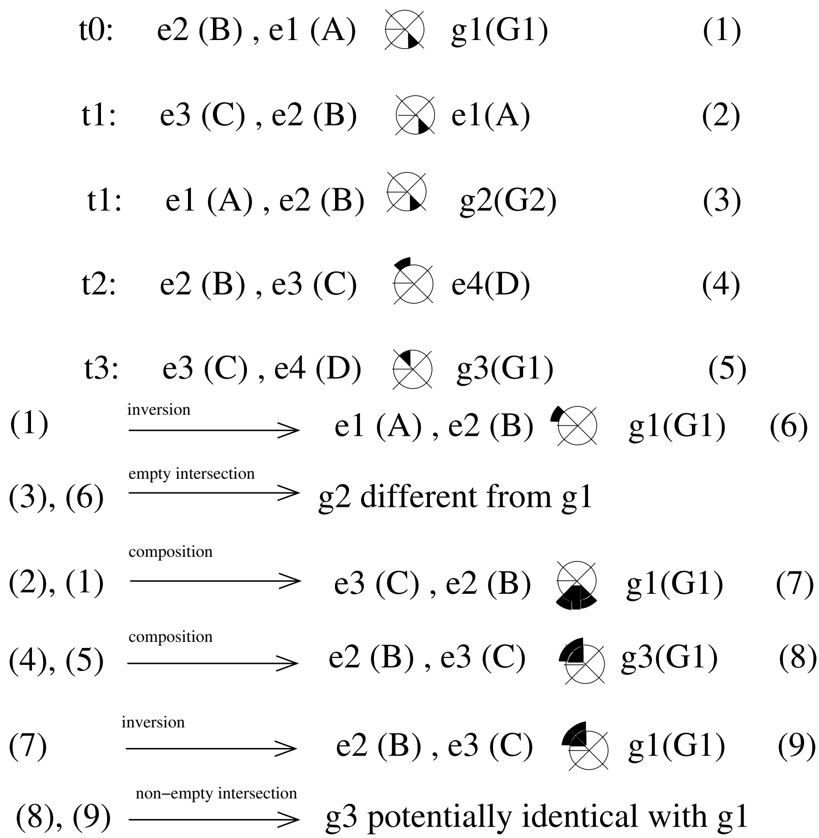
The observed crossing points are denoted $e1, e2, e3, e4$. The corresponding points on figure 10 are appended in brackets. Please note that landmark $G1$ gets a new internal label $g3$ by the exploring agent when observed the second time. Using these observations we make the following inferences on a syntactical basis using the operations defined in section 3:

We apply the inversion tranform to equation (1):

$$e1(A), e2(B) \quad \text{le} \quad g1(G1) \tag{6}$$

Now we test whether $g1$ and $g2$ can be the same landmark. Therefore we make the assumption that $g1$ and $g2$ are the same point. The intersection operation between qualitative spatial relations about the same points is simply the set theoretic intersection about the sets of atomic relations associated with each of the two relations. Since we made the assumption that $g1$ and $g2$ are the same we can apply the intersection operation an equations (3) and (6). The intersection is empty. The empty set as qualitative spatial relation corresponds semantically to an impossible spatial arrangement of points. Then we can deduce a contradiction from our assumption that $g1$ and $g2$ are the same point. It follows that $g2$ is different from $g1$.

For comparism we use the TPCC calculus for the same example. The observations and the inferences are:


t0: e2 (B) , e1 (A) g1(G1) (1)
t1: e3 (C) , e2 (B) e1(A) (2)
t1: e1 (A) , e2 (B) g2(G2) (3)
t2: e2 (B) , e3 (C) e4(D) (4)
t3: e3 (C) , e4 (D) g3(G1) (5)
(1) inversion e1 (A) , e2 (B) g1(G1) (6)
(3), (6) empty intersection g2 different from g1
(2), (1) composition e3 (C) , e2 (B) g1(G1) (7)
(4), (5) composition e2 (B) , e3 (C) g3(G1) (8)
(7) inversion e2 (B) , e3 (C) g1(G1) (9)
(8), (9) non−empty intersection g3 potentially identical with g1